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ON INTERVAL VALUED VAGUE WEAKLY VOLTERRA SPACES

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ABSTRACT

The purpose of this paper is to investigate several characterizations of interval valued vague weakly Volterra space and derive its relations with other spaces.

Keywords: Interval valued vague σ - nowhere dense, interval valued vague σ - baire space, interval valued vague weakly Volterra space, interval valued vague almost resolvable space.

I. INTRODUCTION

In order to deal with uncertainties, the idea of fuzzy set and its operations were introduced by L. A. Zadeh[11] in his classical paper in the year 1965, describing fuzziness mathematically for the first time. The concept of fuzzy topology which was defined by C. L. Chang[1] in the year 1968, paved the way for the subsequent and tremendous growth of the numerous fuzzy topological concepts. Today fuzzy topology has been firmly established as one of the basic disciplines of mathematics. In 1993 Gau and Buehrer [5] introduced the concept of vague set which was the generalization of fuzzy set with truth membership and false membership function. The concept of Volterra spaces have been studied extensively in classical topology in [2],[3],[4] and [7]. The concept of fuzzy Volterra spaces and fuzzy weakly Volterra spaces in fuzzy settings was introduced and studied by G. Thangaraj and S. Soundararajan[10] in 2013. In this paper several characterizations of interval valued vague weakly Volterra spaces are studied and the inter- relations between interval valued vague σ - baire space, interval valued vague almost resolvable space, interval valued vague submaximal spaces, interval valued vague first category spaces, interval valued vague second category spaces are also investigated.

PRELIMINARIES

Definition 2.1: [6] Let [I] be the set of all closed subintervals of the interval [0,1] and $\mu = [\mu_L, \mu_U] \in [I]$, where μ_L and μ_U are the lower extreme and the upper extreme, respectively. For a set X, an IVFS A is given by equation $A = \{< x, \mu_A(x) > / x \in X\}$ where the function $\mu_A : X \to [I]$ defines the degree of membership of an element x to A, and $\mu_A(x) = [\mu_{AL}(x), \mu_{AU}(x)]$ is called an interval valued fuzzy number.

Definition 2.2: [5] A vague set A in the universe of discourse U is characterized by two membership functions given by:

(i) A true membership function $t_A : U \to [0,1]$ and
(ii) A false membership function $f_A : U \to [0,1]$

where $t_A(x)$ is a lower bound on the grade of membership of x derived from the "evidence for x", $f_A(x)$ is a lower bound on the negation of x derived from the "evidence for x", and $t_A(x) + f_A(x) \leq 1$. Thus the grade of membership of u in the vague set A is bounded by a subinterval $[t_A(x), 1 - f_A(x)]$ of [0,1]. This indicates that if the actual grade of membership of x is μ(x), then, $t_A(x) \leq \mu(x) \leq 1 - f_A(x)$.The vague set A is written as $A = \{\langle x, [t_A(x), 1 - f_A(x)]\rangle / u \in U\}$ where the interval $[t_A(x), 1 - f_A(x)]$ is called the vague value of x in A, denoted by $V_A(x)$.

Definition 2.3:[9] An interval valued vague sets $\widetilde{A}^V$ over a universe of discourse X is defined as an object of the form $\widetilde{A}^V = \{< x_i, [T_{\widetilde{A}^V}(x_i), F_{\widetilde{A}^V}(x_i)] >, x_i \in X\}$ where $T_{\widetilde{A}^V} : X \to D([0,1])$ and $F_{\widetilde{A}^V} : X \to D([0,1])$ are called " truth membership function" and "false membership function" respectively and where D[0,1] is the set of all intervals within [0,1], or in other word an interval valued vague set can be represented by $\widetilde{A}^V =< [(x_i),[\mu_1,\mu_2],[v_1,v_2]] >, x_i \in X$ where $0 \le \mu_1 \le \mu_2 \le 1$ *and* $0 \le v_1 \le v_2 \le 1$. For each interval valued vague set $\widetilde{A}^V$, $\pi_{1\widetilde{A}^V}(x_i) = 1 - \mu_{1\widetilde{A}^V}(x_i) - v_{1\widetilde{A}^V}(x_i)$ are called degree of hesitancy of x_i in $\widetilde{A}^V$ respectively.

Definition 2.4:[8] An interval valued vague topology (IVT in short) on X is a family τ of interval valued vague sets(IVS) in X satisfying the following axioms.

(i) 0, 1 $\in \tau$

(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$

(iii) $\bigcup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X,τ) is called an interval valued vague topological space (IVTS in short) and any IVS in τ is known as a Interval valued vague open set(IVOS in short) in X.

The complement $\overline{A}$ of a IVOS A in a IVTS (X, τ) is called an interval valued vague closed set (IVCS in short) in X.

Definition 2.5:[8] Let $A = \{\langle x, [t_A^L(x), t_A^U(x)], [1 - f_A^L(x), 1 - f_A^U(x)]\rangle\}$ and $B = \{\langle x, [t_B^L(x), t_B^U(x)], [1 - f_B^L(x), 1 - f_B^U(x)]\rangle\}$ be two interval valued vague sets then their union, intersection and complement are defined as follows:

(i) $A \cup B = \{< x, [t_{A\cup B}^L(x), t_{A\cup B}^U(x)], [1 - f_{A\cup B}^L(x), 1 - f_{A\cup B}^U(x)] > / x \in X\}$ where

$$t_{A\cup B}^L(x) = \max\{t_A^L(x), t_B^L(x)\},\ t_{A\cup B}^U(x) = \max\{t_A^U(x), t_B^U(x)\}\ \textit{and}$$
$$1 - f_{A\cup B}^L(x) = \max\{1 - f_A^L(x), 1 - f_B^L(x)\},\ 1 - f_{A\cup B}^U(x) = \max\{1 - f_A^U(x), 1 - f_B^U(x)\}$$

(ii) $A \cap B = \{< x, [t_{A\cap B}^L(x), t_{A\cap B}^U(x)], [1 - f_{A\cap B}^L(x), 1 - f_{A\cap B}^U(x)] > / x \in X\}$ where

$$t_{A\cap B}^L(x) = \min\{t_A^L(x), t_B^L(x)\},\ t_{A\cap B}^U(x) = \min\{t_A^U(x), t_B^U(x)\}\ \textit{and}$$
$$1 - f_{A\cap B}^L(x) = \min\{1 - f_A^L(x), 1 - f_B^L(x)\},\ 1 - f_{A\cap B}^U(x) = \min\{1 - f_A^U(x), 1 - f_B^U(x)\}$$

(iii) $\overline{A} = \{< x, [f_A^L(x), f_A^U(x)], [1 - t_A^L(x), 1 - t_A^U(x)] > / x \in X\}$.

Definition 2.6:[8] Let (X, τ) be an interval valued vague topological space and $A = \{\langle x, [t_A^L, t_A^U], [1 - f_A^L, 1 - f_A^U]\rangle\}$ be a IVS in X. Then the interval valued vague interior and an interval valued vague closure are defined by

$IV\operatorname{int}(A)$ = $\bigcup$ {G/G is an IVOS in X and G $\subseteq$ A}

$IVcl(A)$ = $\bigcap$ {K/K is an IVCS in X and A $\subseteq$ K}

Note that for any IVS A in (X,τ), we have $IVcl(\overline{A}) = \overline{IV\operatorname{int}(A)}$ and $V\operatorname{int}(\overline{A}) = \overline{IVcl(A)}$. and $IVcl(A)$ is an IVCS and $IV\operatorname{int}(A)$ is an IVOS in X. Further we have, if A is an IVCS in X thenIVcl(A)=A and if A is an IVOS in X then IVint(A)=A.

Definition 2.7:[8] An interval valued vague set A in an interval valued vague topological space (X,τ) is called an interval valued vague dense if there exists no interval valued vague closed set B in (X,τ) such that $A \subset B \subset 1$.

Definition 2.8:[8] An interval valued vague set A in an interval valued vague topological space (X,τ) is called an interval valued vague nowhere dense set if there exists no interval valued vague open set B in (X,τ) such that $B \subset IVcl(A)$. That is, $IV\,\text{int}(IVcl(A)) = 0$.

Theorem 2.9:[8] If A is an interval valued vague dense and interval valued vague open set in an interval valued vague topological space (X,τ) then A^c is a interval valued vague nowhere dense set in (X,τ).

Definition 2.10:[8] An interval valued vague topological space (X,τ) is called an interval valued vague first category set if $A = \bigcup_{i=1}^{\infty}(A_i)$, where A_i's are interval valued vague nowhere dense sets in (X,τ). Any other interval valued vague set in (X,τ) is said to be of interval valued vague second category.

Definition 2.11:[8] An interval valued vague set A in an interval valued vague topological space (X,τ) is called an interval valued vague G_δ-sets in (X,τ) if $A = \bigcap_{i=1}^{\infty}(A_i)$ where $A_i \in \tau$, for $i \in I$.

Definition 2.12:[8] An interval valued vague set A in an interval valued vague topological space (X,τ) is called an interval valued vague F_σ-sets in (X,τ) if $A = \bigcup_{i=1}^{\infty}(A_i)$ where $\overline{A_i} \in \tau$, for $i \in I$.

Definition 2.13:[8] An interval valued vague topological space (X,τ) is called an interval valued vague Volterra space if $IVcl(\bigcap_{i=1}^{N} A_i) = 1$, where A_i's are interval valued vague dense and interval valued vague G_δ-sets in (X,τ).

Definition 2.14:[8] Let (X,τ) be an interval valued vague topological space. Then (X,τ) is called an interval valued vague baire space if $IV\,\text{int}(\bigcup_{i=1}^{\infty} A_i) = 0$ where A_i's are interval valued vague nowhere dense sets in (X,τ).

3. Interval valued vague weakly Volterra spaces:

Definition 3.1: Let (X,τ) be an interval valued vague topological space. An interval valued vague set A in (X,τ) is called an interval valued vague σ- nowhere dense set if A is an interval valued vague F_σ set in (X,τ) such that $IV\,\text{int}(A) = 0$.

Definition 3.2: Let (X,τ) be an interval valued vague topological spaces. Then (X,τ) is called an interval valued vague σ - Baire space if $IV\,\text{int}(\bigcup_{i=1}^{\infty} A_i) = 0$, where A_i's are interval valued vague σ - nowhere dense set (X,τ).

Theorem 3.3: In an interval valued vague topological space (X,τ) an interval valued vague set A is an Interval valued vague σ - nowhere dense set in (X,τ) if and only if A^c is an interval valued vague dense and interval valued vague G_δ-set in (X,τ).

Proof: Let A be an interval valued vague σ - nowhere dense set in (X,τ). Then $A = \bigcup_{i=1}^{\infty}(A_i)$ where $A_i^c \in \tau$, for $i \in I$ and $IV\,\text{int}(A) = 0$. Then $(IV\,\text{int}(A))^c = (0)^c = 1$ implies that $IVcl(A^c) = 1$. Also

$A^c = (\bigcup_{i=1}^{\infty}(A_i))^c = \bigcap_{i=1}^{\infty}(A_i^c)$ where $A_i^c \in \tau$,for $i \in I$. Hence we have A^c is an interval valued vague dense and interval valued vague G_δ-set in (X,τ).

Conversely, let A be an interval valued vague dense and interval valued vague G_δ-set in (X,τ). Then $A = \bigcap_{i=1}^{\infty}(A_i)$ where $A_i \in \tau$,for $i \in I$. Now $A^c = (\bigcap_{i=1}^{\infty}(A_i))^c = \bigcup_{i=1}^{\infty}(A_i^c)$. Hence A^c is an interval valued vague F_σ set in (X,τ) and since A is an interval valued vague dense set we have $(IV\operatorname{int}(A^c)) = 0$ .Therefore A^c is an interval valued vague σ - nowhere dense set in (X,τ).

Theorem 3.4: If A is an Interval valued vague dense set in (X,τ) such that $B \subseteq A^c$, where B is an interval valued vague F_σ set in (X,τ) .Then B is an interval valued vague vague σ - nowhere dense set in (X,τ).

Proof: Let A be an interval valued vague dense set in (X,τ) such that $B \subseteq A^c$ .Now $B \subseteq A^c$ implies that $IV\operatorname{int}(B) \subseteq IV\operatorname{int}(A^c) = (IVcl(A))^c = 0$ and hence $IV\operatorname{int}(B) = 0$. Therefore B is an interval valued vague σ - nowhere dense set in (X,τ).

Theorem 3.5: If A is an interval valued vague F_σ set and interval valued vague nowhere dense set in (X,τ) ,then A is an interval valued vague σ - nowhere dense set in (X,τ).

Proof: Now $A \leq IVcl(A)$ for any interval valued vague set in (X,τ). Then, $IV\operatorname{int}(A) \leq IV\operatorname{int}(IVcl(A))$. Since A is an interval valued vague nowhere dense set in (X,τ), $IV\operatorname{int}(IVcl(A)) = 0$ and hence $IV\operatorname{int}(A) = 0$ and A is an interval valued vague F_σ set implies that A is an interval valued vague σ - nowhere dense set in (X,τ).

Theorem 3.6: If A_i's (i=1,2,….N) are interval valued vague σ - nowhere dense set in (X,τ) and $IV\operatorname{int}(\bigcup_{i=1}^{N} A_i) = 0$, then (X,τ) is an interval valued vague Volterra space.

Proof: Let A_i's (i=1,2,….N) are interval valued vague σ - nowhere dense set in (X,τ) then A_i's are interval valued vague F_σ set with $IV\operatorname{int}(A_i) = 0$. Now $(IV\operatorname{int}(A_i))^c = 1$. Then, we have $IVcl(A_i^c) = 1$. That is, A_i^c's are interval valued vague dense set in (X,τ). Since A_i's are interval valued vague F_σ set, A_i^c's are interval valued vague G_δ-sets in (X,τ). Hence A_i^c's are interval valued vague dense and interval valued vague G_δ-sets in (X,τ). Now $IVcl(\bigcap_{i=1}^{N}(A_i^c)) = (IV\operatorname{int}(\bigcup_{i=1}^{N} A_i))^c = 0^c = 1$. Hence (X,τ) is an interval valued vague Volterra space

Definition 3.7: An interval valued vague topological space (X,τ) is called an interval valued vague weakly Volterra space if $IVcl(\bigcap_{i=1}^{N} A_i) \neq 0$, where A$_i$'s are interval valued vague dense and interval valued vague G_δ-set in (X,τ).

Example 3.8: Let X={a,b}. The interval valued vague sets are defined as follows $A = \{x, [[0.3,0.4],[0.6,0.8]], [[0.3,0.4],[0.6,0.9]]\}$, $B = \{x, [[0.3,0.5],[0.6,0.7]], [[0.2,0.3],[0.4,0.5]]\}$,

$C=\{<x,[[0.3,0.4],[0.6,0.7]],[[0.1,0.3],[0.4,0.5]]>\}$ and $D=\{<x,[[0.3,0.5],[0.6,0.8]],[[0.3,0.4],[0.6,0.9]]>\}$. Clearly $\tau=\{0,1,A,B,C,D\}$ is an interval valued vague topology in X. Thus (X,τ) is an interval valued vague topological space. $IVcl(A)=1,\ IVcl(B)=1,\ IVcl(D)=1$. Now $IVcl(A\bigcap B\bigcap D)\neq 0$. Therefore (X,τ) is an interval valued vague weakly Volterra space but it is not an interval valued vague Volterra space.

Definition 3.9: Let (X,τ) be an interval valued vague topological space. An interval valued vague set A in (X,τ) is called interval valud vague σ - first category if $A=\bigcup_{i=1}^{\infty}A_i$ where A_i's are interval valued vague σ - nowhere dense set in (X,τ). Any other interval valued vague set in (X,τ) is said to be interval valued vague σ - second category in (X,τ).

Definition 3.10: An interval valued vague topological space (X,τ) is an interval valued vague σ - first category space if $1=\bigcup_{i=1}^{\infty}A_i$, where A_i's are interval valued vague σ - nowhere dense set in (X,τ). (X,τ) is called an interval valued vague vague σ - second category space if it is not an interval valued vague σ - first category space.

Theorem 3.11: If the interval valued vague topological space (X,τ) is an interval valued vague σ - second category space, then (X,τ) is an interval valued vague weakly Volterra space.

Proof: Let A_i's (i=1,2,….,N) be interval valued vague dense and interval valued vague G_δ-set in (X,τ). Then by Theorem: 3.3 A_i^c's are interval valued vague σ - nowhere dense set in (X,τ). Let B_α $(\alpha=1,2,\ldots\ldots,\infty)$ be an interval valued vague σ - nowhere dense set in (X,τ) in which let us take the first $N(B_\alpha)'s$ as A_i^c. Since (X,τ) is an interval valued vague σ - second category space, $\bigcup_{\alpha=1}^{\infty}B_\alpha\neq 1$. Then $(\bigcup_{\alpha=1}^{\infty}B_\alpha)^c\neq 1^c \Rightarrow \bigcap_{\alpha=1}^{\infty}B_\alpha^c\neq 0$. Then we have $IVcl(\bigcap_{\alpha=1}^{\infty}(B_\alpha)^c)\neq 0$. Since $IVcl(\bigcap_{\alpha=1}^{N}(B_\alpha)^c)\leq IVcl(\bigcap_{\alpha=1}^{\infty}(B_\alpha)^c)$, then we have $IVcl(\bigcap_{\alpha=1}^{N}(B_\alpha)^c)\neq 0$, where A_i's (i=1,2,….,N) are interval valued vague dense and interval valued vague G_δ-set in (X,τ). Therefore (X,τ) is an interval valued vague weakly Volterra space.

Theorem 3.12:

(i) Let (X,τ) be an interval valued vague weakly Volterra space and if $\bigcup_{i=1}^{N}(A_i)=1$, where A_i's are interval valued vague F_σ-set in (X,τ) then there exists atleast one A_i in (X,τ) with $IV\operatorname{int}(A_i)\neq 0$.

(ii) If $\bigcup_{i=1}^{N}(A_i)=1$ where A_i's are interval valued vague F_σ-set in (X,τ) and if, $IV\operatorname{int}(A_i)\neq 0$ for atleast one (i=1,2,…N) then (X,τ) is an interval valued vague weakly Volterra space.

Proof: (i)$\Rightarrow$(ii) Suppose that $IV\operatorname{int}(A_i)=0$; for all i=1,2,…,N. Then $(IV\operatorname{int}(A_i))^c=1\Rightarrow IVcl(A_i)^c=1$. Therefore A_i^c's are interval valued vague dense set in X. A_i's are interval valued vague F_σ-set in (X,τ) implies that A_i^c's are interval valued vague G_δ-set. Now , $IVcl(\bigcap_{i=1}^{N}(A_i^c)) = IVcl(\bigcup_{i=1}^{N}A_i)^c = IVcl((1)^c) = 0$. Therefore, $IVcl(\bigcap_{i=1}^{N}(A_i^c)) = 0$, where A_i^c's are interval valued vague dense set and interval valued vague G_δ-set. This implies (X,τ) is not an interval valued vague weakly Volterra space, which is a contradiction. Therefore $IV\operatorname{int}(A_i) \neq 0$ for atleast one i (i=1,2,….,N) in (X,τ).

(ii)$\Rightarrow$(i) Suppose that (X,τ) is not an interval valued vague weakly Volterra space. $IVcl(\bigcap_{i=1}^{N}A_i) = 0$ where A_i's are interval valued vague dense and interval valued vague G_δ-set in (X,τ). This implies that $IV\operatorname{int}(\bigcup_{i=1}^{N}(A_i)^c) = 1 \Rightarrow \bigcup_{i=1}^{N}(A_i)^c = 1$, where A_i^c's are interval valued vague F_σ-set in (X,τ) and $IV\operatorname{int}(A_i)^c = 0$ (because $IVcl(A_i) = 1\ \forall\ i = 1,2,....., N$) which is a contradiction to the hypothesis. Hence, (X,τ) must be an interval valued vague weakly Volterra space.

Definition 3.13: An interval valued vague topological space (X,τ) is an interval valued vague almost resolvable space if $\bigcup_{i=1}^{\infty}A_i = 1$, where the interval valued vague set, A_i's in (X,τ) are such that $IV\operatorname{int}(A_i) = 0$. Otherwise, (X,τ) is called an interval valued vague almost irresolvable.

Definition 3.14: An interval valued vague topological space (X,τ) is called an interval valued vague p-space if countable intersection of interval valued vague open sets in (X,τ) is an interval valued vague open in (X,τ).

Definition 3.15: An interval valued vague topological space (X,τ) is called an interval valued vague submaximal space if for each interval valued vague set A in (X,τ) such that $IVcl(A) = 1$, then $A \in \tau$.

Theorem 3.16: If the interval valued vague topological space (X,τ) is an interval valued vague almost irresolvable space, then (X,τ) is an interval valued vague weakly Volterra space.

Proof: Let A_i's (i=1,2,….,N) be interval valued vague dense and interval valued vague G_δ-set in (X,τ). Now $IVcl(A_i) = 1 \Rightarrow IV\operatorname{int}(A_i)^c = 0$. Since (X,τ) is an interval valued vague almost irresolvable space, $\bigcup_{i=1}^{\infty}B_i \neq 1$, where the interval valued vague sets B_i's in (X,τ) are such that $IV\operatorname{int}(B_i) = 0$. Let us take the first $N(B_i)$'s as $(A_i)^c$'s in (X,τ). Now, $\bigcup_{i=1}^{\infty}B_i \neq 1 \Rightarrow (\bigcup_{i=1}^{\infty}B_i)^c \neq 0$. This implies that $\bigcap_{i=1}^{\infty}(B_i)^c \neq 0$ and thus $IVcl(\bigcap_{i=1}^{\infty}(B_i)^c) \neq 0$. Since $IVcl(\bigcap_{i=1}^{N}(B_i)^c) \leq IVcl(\bigcap_{i=1}^{\infty}(B_i)^c)$ then $IVcl(\bigcap_{i=1}^{N}(B_i)^c) \neq 0$. Hence $IVcl(\bigcap_{i=1}^{N}((A_i)^c)^c) \neq 0$ replacing B_i, by $(A_i)^c$, i=1, 2, …,N. This implies $IVcl(\bigcap_{i=1}^{N}(A_i)) \neq 0$. Therefore (X,τ) is an interval valued vague weakly Volterra space.

Theorem 3.17: If the interval valued vague topological space (X,τ) is an interval valued vague second category and interval valued vague p- space, then (X,τ) is an interval valued vague weakly Volterra space.

Proof: Let A_i's (i=1,2,….,N) be interval valued vague dense and interval valued vague G_δ-set in (X,τ). Since (X,τ) is an interval valued vague p- spaces then interval valued vague G_δ-set A_i's are interval valued vague open set in (X,τ). Then A_i's (i=1,2,….,N) be interval valued vague dense and interval valued vague open set in (X,τ). Then by theorem 2.9, A_i^c's are interval valued vague nowhere dense set in (X,τ). Since (X,τ) is an interval valued vague second category space $\bigcup_{i=1}^{\infty} B_i \neq 1$ where $B_i's$ are interval valued vague nowhere dense set in (X,τ).Let us take the first $N(B_i)'s$ as $(A_i)^c's$ in (X,τ). Then, $\bigcup_{i=1}^{N}(A_i)^c = \bigcup_{i=1}^{N} B_i \subseteq \bigcup_{i=1}^{\infty} B_i \neq 1$. This implies that, $(\bigcup_{i=1}^{N}(A_i)^c)^c \neq 0 \Rightarrow \bigcap_{i=1}^{N} A_i \neq 0$. Thus $IVcl(\bigcap_{i=1}^{N} A_i) \neq 0$, where A_i's are interval valued vague dense and interval valued vague G_δ-set in (X,τ). So (X,τ) is an interval valued vague weakly Volterra space.

Theorem 3.18: If the interval valued vague topological space (X,τ) is an interval valued vague second category and interval valued vague submaximal space, then (X,τ) is an interval valued vague weakly Volterra space.

Proof: Let A_i's (i=1,2,….,N) be interval valued vague dense and interval valued vague G_δ-set in (X,τ). Since (X,τ) is interval valued vague submaximal space, the interval valued vague dense set A_i's are interval valued vague open set in (X,τ). By theorem 2.9, A_i^c's are interval valued vague nowhere dense sets in (X,τ). Since (X,τ) is an interval valued vague second category space $\bigcup_{i=1}^{\infty} B_i \neq 1$ where $B_i's$ are interval valued vague nowhere dense set in (X,τ).Let us take the first $N(B_i)'s$ as $(A_i)^c's$ in (X,τ). Then, $\bigcup_{i=1}^{N}(A_i)^c \leq \bigcup_{i=1}^{\infty} B_i$ and $\bigcup_{i=1}^{\infty} B_i \neq 1$, implies that $\bigcup_{i=1}^{N}(A_i)^c \neq 1$ This implies that, $\bigcap_{i=1}^{N} A_i \neq 0$ and hence $IVcl(\bigcap_{i=1}^{N} A_i) \neq 0$, where A_i's are interval valued vague dense and interval valued vague G_δ-set in (X,τ). Therefore (X,τ) is an interval valued vague weakly Volterra space.

Theorem 3.19: If the interval valued vague topological space (X,τ) is not an interval valued vague weakly Volterra space, then (X,τ) is an interval valued vague σ - first category space.

Proof: Let $B_i's$ $(i=1,2,......,\infty)$ be an interval valued vague σ - nowhere dense sets in an interval valued vague topological space (X,τ) which is not an interval valued vague weakly Volterra space. Now, we claim that $\bigcup_{i=1}^{\infty} B_i = 1$. Suppose that $\bigcup_{i=1}^{\infty} B_i \neq 1$. Then $\bigcap_{i=1}^{\infty}(B_i)^c \neq 0$. Since $B_i's$ are interval valued vague σ - nowhere dense set in (X,τ) by theorem 3.3, $(B_i)^c's$ are interval valued vague dense and interval valued vague G_δ-set in (X,τ). Now, $\bigcap_{i=1}^{N}(B_i)^c \subseteq \bigcap_{i=1}^{\infty}(B_i)^c$ implies that $\bigcap_{i=1}^{N}(B_i)^c \neq 0$, Let $A_i = (B_i)^c$, then $\bigcap_{i=1}^{N}(A_i) \neq 0$ implies

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that $IVcl(\bigcap_{i=1}^{N}(A_i)) \neq 0$, where A_i's are interval valued vague dense and interval valued vague G_δ-set in (X,τ). But this is a contradiction, since (X,τ) is not an interval valued vague weakly Volterra space. Hence $\bigcup_{i=1}^{\infty} B_i = 1$. Therefore, (X,τ) is an interval valued vague σ - first category space.

Theorem 3.20: If an interval valued vague topological space (X,τ) is an interval valued vague weakly Volterra space, then (X,τ) is not an interval valued vague σ - baire space.

Proof: Let (X,τ) be an interval valued vague weakly Volterra space. Then, we have $IVcl(\bigcap_{i=1}^{N} A_i) \neq 0$, where A_i's are interval valued vague dense and interval valued vague G_δ-set in (X,τ). Since A_i's are interval valued vague dense and interval valued vague G_δ-set in (X,τ), then by theorem 3.3 . Let B_i's $(i = 1,2,.....\infty)$ be an interval valued vague σ - nowhere dense set in (X,τ) in which the first N interval valued vague σ - nowhere dense set be A_i^{c}'s. Now $\bigcup_{i=1}^{N}(A_i^{c}) \leq \bigcap_{i=1}^{\infty} B_i$. Then $IV\,\mathrm{int}(\bigcup_{i=1}^{N}(A_i^{c})) \leq IV\,\mathrm{int}(\bigcap_{i=1}^{\infty} B_i)$ this implies that $(IVcl(\bigcap_{i=1}^{N} A_i))^{c} \leq IV\,\mathrm{int}(\bigcup_{i=1}^{\infty} B_i)$. Since $IVcl(\bigcap_{i=1}^{N} A_i) \neq 0$, $IV\,\mathrm{int}(\bigcup_{i=1}^{\infty} B_i) \neq 0$, where B_i's $(i = 1,2,.....\infty)$ are interval valued vague σ - nowhere dense set (X,τ). Hence (X,τ) is not an interval valued vague σ - baire space.

Remark: The interrelations between interval valued vague weakly Volterra space and other spaces are summarized in the following diagram:

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